MCA11A

Reg. No. | | | | | | | | | | |

First Semester M.C.A Degree Examination, January/February 2003

Master of Computer Applications

New Scheme

Mathematics

Time: 3 hrs.] [Max.Marks : 100

Note: *1. Answer any FIVE full questions choosing at least TWO questions from each part.*
2. All questions carry equal marks.

PART - A

1. (a) Prove that
$(1 + cos\theta + i\ sin\theta)^n + (1 + cos\theta - i\ sin\theta)^n = 2^{n+1}cos^n(\theta/2)cos(n\theta/2)$ **(6 Marks)**

(b) Expand $cos^8\theta$ in a series of cosines of multiples of θ. **(7 Marks)**

(c) Separate $tan^{-1}(x + iy)$ into real and imaginary parts. **(7 Marks)**

2. (a) If cosh (u+iv) = x+iy, prove that
$x^2sec^2V - y^2cosec^2v = 1$. **(6 Marks)**

(b) State De Moivre's theorem. Use it to solve the equation $x^4 - x^3 + x^2 - x + 1 = 0$. **(7 Marks)**

(c) Prove that $\log\left(\frac{a+ib}{a-ib}\right) = 2itan^{-1}(b/a)$.
Hence evaluate $cos\left[ilog(\frac{a+ib}{a-ib})\right]$ **(7 Marks)**

3. (a) Using elementary row operations find the rank of the matrix.

$$\begin{pmatrix} 0 & 1 & -3 & -1 \\ 0 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{pmatrix}$$

(6 Marks)

(b) Find the eigen values and eigen vector corresponding to least eigen value of the matrix.

$$\begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$$

(7 Marks)

(c) Use Cayley- Hamilton theorem to find the inverse of the matrix.

$$\begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{pmatrix}$$

(7 Marks)

Contd.... 2

4. (a) Define convergence and divergence of a series of positive terms. Also, test for the convergence of
$\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \ldots.$ (6 Marks)

(b) Discuss the convergence of the series
$\frac{1}{2\sqrt{1}} + \frac{x^2}{3\sqrt{2}} + \frac{x^4}{4\sqrt{3}} + \frac{x^6}{5\sqrt{4}} + \ldots.$ (7 Marks)

(c) State Leibnitz's rule for alternating series. Test whether the series
$\frac{1}{1^2} + \frac{1}{2^2} - \frac{1}{3^2} + \frac{1}{4^2} - \frac{1}{5^2} + \ldots..$
is convergent or not? (7 Marks)

PART - B

5. (a) Find the first derivative of y where
$y = x^{tanx} + (sinx)^{cosx}$. (5 Marks)

(b) Find the n^{th} derivative of $\frac{x}{(x-1)(2x+3)}$ (5 Marks)

(c) Find the angle of intersection of the curves
$r = a(1 - cos\theta)$ and $r = b(1 + cos\theta)$. (5 Marks)

(d) Evaluate : $\lim_{x \to 0}\left(\frac{1}{sinx} - \frac{1}{x}\right)$ (5 Marks)

6. (a) If Z = f(x+ct) + g(x-ct), prove that
$\frac{\partial^2 z}{\partial t^2} = c^2 \frac{\partial^2 z}{\partial x^2}$ (6 Marks)

(b) State Euler's theorem for a function of two variables. Use it show that
$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = sin2u$
Where $u = tan^{-1}\{(x^3 + y^3)/x + y\}$ (7 Marks)

(c) If $u = f(x - y, y - z, z - x)$, prove that
$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0.$ (7 Marks)

7. (a) Evaluate :
i) $\int \frac{x^2+1}{x^4+1}dx$ ii) $\int \frac{dx}{5+4cosx}$ (6 Marks)

(b) Obtain a reduction formula for $\int_0^{\pi/2} cos^n x dx (n - even)$ and hence evaluate
$\int_0^{\pi/2} cos^6 x dx$ (7 Marks)

(c) Evaluate : $\int_0^1 \int_{x^2}^{2-x} xy dx dy$ (7 Marks)

8. (a) Solve : $\frac{dy}{dx} = (x^2 - y^2)/2xy$ (6 Marks)

(b) Solve : $\frac{dy}{dx} + \frac{ycosx+siny+y}{sinx+xcosy+x} = 0$ (7 Marks)

(c) Solve : $\frac{dy}{dx} + ycotx = cosx$ (7 Marks)

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NEW SCHEME

MCA11A

USN | | | | | | | | | | |

First Semester M.C.A Degree Examination, July/August 2003

Master of Computer Applications

(New Scheme)

Mathematics

Time: 3 hrs.] [Max.Marks : 100

Note: *1. Answer any FIVE full questions choosing at least TWO questions from each part.*

PART - A

1. (a) If $A + B = 45^0$, show that $(1 + tanA)(1 + tanB) = 2$. Hence deduce the value of tan $22\frac{1}{2}^0$. **(7 Marks)**

(b) If $tan\alpha = \frac{1-cosB}{sinB}$, prove that $tan2\alpha = tanB$. **(6 Marks)**

(c) If $sin\alpha + sin\beta + sin\gamma = cos\alpha + cos\beta + cos\gamma = 0$, prove that $sin3\alpha + sin3\beta + sin3\gamma = 3sin(\alpha + \beta + \gamma)$ and $cos3\alpha + cos3\beta + cos3\gamma = 3cos(\alpha + \beta + \gamma)$. **(7 Marks)**

2. (a) If θ_1, θ_2, θ_3 be 3 values of θ, which satisfy the equation $tan2\theta = \lambda\ tan(\theta + \alpha)$ and such that no two of them differ by a multiple of π, show that $\theta_1 + \theta_2 + \theta_3 + \alpha$ is a multiple of π. **(7 Marks)**

(b) If $sin(A + ib) = x + iy$, prove that

$$\frac{x^2}{cosh^2B} + \frac{y^2}{sinh^2B} = 1 \text{ and } \frac{x^2}{sin^2A} - \frac{y^2}{cos^2A} = 1$$ **(7 Marks)**

(c) Show that

i) $log(1 + itan\alpha) = log(sec\alpha) + i\alpha$, where α is an acute angle.

ii) $Loge\frac{3-i}{3+i} = 2i(n\pi - tan^{-1}\frac{1}{3})$ **(6 Marks)**

3. (a) Test for consistency and solve the system of equations :

$$3x_1 + 2x_2 + 4x_3 = 7$$
$$2x_1 + x_2 + x_3 = 4$$
$$x_1 + 3x_2 + 5x_3 = 2$$

(6 Marks)

(b) Find the eigen values and eigen vectors of the matrix

$$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$ **(7 Marks)**

(c) Use Caley - Hamilton theorem to find the inverse of the matrix.

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$$ **(7 Marks)**

Contd.... 2

4. (a) Test the convergence of the series
$$\sum_{n=1}^{\infty}\left(\sqrt{n^2+1}-n\right)$$ **(6 Marks)**

(b) Test the convergence of the series
$$a+\frac{3}{7}x+\frac{3}{7}\frac{6}{10}x^2+\frac{3}{7}\frac{6}{10}\frac{9}{13}x^3+\ldots\ldots$$ **(7 Marks)**

(c) Test for absolute and conditional convergence
$$1-\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}-\frac{1}{\sqrt{4}}+\ldots\ldots$$ **(7 Marks)**

PART - B

5. (a) i) A function f is defined in (0,3) as follows

$f(x) = x^2$ when $0 < x < 1$
$= x$ when $1 \le x < 2$
$= \frac{x^3}{4}$ when $2 \le x < 3$

Examine the continuity of the function at $x = 2$.

ii) Find $\frac{dy}{dx}$, $if\ x^y = y^x$. **(3+3 Marks)**

(b) If $y = acos(logx) + 6sin(logx)$, prove that
$$x^2y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0.$$ **(7 Marks)**

(c) Show that the curves $r^m = a^m cosm\theta$ and $r^m = a^m sinm\theta$ cut each other orthogonally. **(7 Marks)**

6. (a) Evaluate $\lim_{x\to o}(cosx)^{\frac{1}{x^2}}$ **(6 Marks)**

(b) If $u = sin^{-1}\frac{x^2+y^2}{x+y}$, show that
$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = tanu.$$ **(7 Marks)**

(c) If $H = f(y-z, z-x, x-y)$, prove that
$$\frac{\partial H}{\partial x} + \frac{\partial H}{\partial y} + \frac{\partial h}{\partial z} = 0$$ **(7 Marks)**

7. (a) i) Evaluate : $\int_0^{\infty}\frac{1}{(a^2+x^2)^2}dx$

ii) Evaluate : $\int x^2 sinx.cosx\,dx$ **(3+4 Marks)**

(b) Evaluate : $\int_0^a x^4\sqrt{a^2-x^2}\,dx$ **(6 Marks)**

(c) Evaluate : $\int_0^5\int_0^{x^2}(x^2+y^2)dxdy.$ **(7 Marks)**

8. (a) Solve : $xydx - (x^2+3y^2)dy = 0$ **(7 Marks)**

(b) Solve : $(x^2+y^2+1)dx + x(x-2y)dy = 0$ **(7 Marks)**

(c) Solve : $y^1 + ytanx = sin2x,\ y(0) = 1$ **(6 Marks)** **(6 Marks)**

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NEW SCHEME

MCA11A

USN

First Semester M.C.A Degree Examination, January/February 2004

Master of Computer Applications

(New Scheme)

Mathematics

Time: 3 hrs.] [Max.Marks : 100

Note: *1. Answer any FIVE full questions choosing at least TWO questions from each part.*
2. All questions carry equal marks.

PART - A

1. (a) Expand $sin^7\theta cos^3\theta$ in a series of sines of multiples of θ **(7 Marks)**

(b) Separate $sec(x + iy)$ into real and imaginary parts. **(7 Marks)**

(c) If $x + 1/x = 2cos\theta$ and $y + 1/y = 2cos\phi$, prove that one of the values of $x^m/y^n + y^n/x^m$ is $2cos(m\theta - n\phi)$. **(6 Marks)**

2. (a) Using De Moivre's theorem, solve $x^7 + x^4 + x^3 + 1 = 0$ **(7 Marks)**

(b) If $\tan\log(x + iy) = a + ib$ and $a^2 + b^2 \neq 1$, prove that
$\tan\log(x^2 + y^2) = 2a/(1 - a^2 - b^2)$ **(7 Marks)**

(c) If $sin(A + iB) = x + iy$, prove that
$x^2cosec^2A - y^2sec^2A = 1$ **(6 Marks)**

3. (a) Find the eigen values and eigen vector corresponding to largest eigen value of the matrix:

$$\begin{pmatrix} -2, & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix}$$ **(7 Marks)**

(b) Use Cayley-Hamilton theorem to find the inverse of the matrix:

$$\begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$$ **(7 Marks)**

(c) Find the rank of the matrix

$$\begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{pmatrix}$$

using elementary row operations. **(6 Marks)**

4. (a) Examine the convergence of the series

$\frac{1}{2} + \frac{1.3}{2.4} + \frac{1.3.5}{2.4.6} + ...$ **(7 Marks)**

(b) Define the terms absolute convergence and conditional convergence. Test whether the series

$2 - \frac{3}{2} + \frac{4}{3} - \frac{5}{4} + ...$

is convergent or not. **(7 Marks)**

Contd.... 2

(c) Test for the convergence or divergence of the series

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n} + \sqrt{n+1}}$$

(6 Marks)

PART B

5. (a) If $y = x^x + (tanx)^{cotx}$, find $(\frac{dy}{dx})$. **(7 Marks)**

(b) Find the angle between the curves:
$r = 2sin\theta$ and $r = sin\theta + cos\theta$ **(7 Marks)**

(c) Find the n^{th} derivative of $e^{4x} sin^3 x$ **(6 Marks)**

6. (a) If $u = x^2 tan^{-1}(y/x) - y^2 tan^{-1}(x/y)$, find the value of $\frac{\partial^2 u}{\partial x \partial y}$ at $x = 2$ and $y = 3$ **(7 Marks)**

(b) If $u = f(x, y)$ where $x = rcos\theta$ and $y = rsin\theta$, prove that
$(\frac{\partial u}{\partial x})^2 + (\frac{\partial u}{\partial y})^2 = (\frac{\partial u}{\partial r})^2 + \frac{1}{r^2}(\frac{\partial u}{\partial \theta})^2$ **(7 Marks)**

(c) Using Euler's theorem for homogeneous functions, show that
$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = tanz$
where $z = sin^{-1}(\frac{x^2+y^2}{x+y})$ **(6 Marks)**

7. (a) Evaluate $\int_A \int xy dx dy$, where A is the domain bounded by x-axis, ordinate $x = 2a$ and the curve $x^2 = 4ay$ **(7 Marks)**

(b) Obtain a reduction formula for $\int_0^{\pi/2} sin^n x dx (n - odd)$ and hence evaluate $\int_0^{\pi/2} sin^5 x dx$ **(7 Marks)**

(c) Evaluate:

i) $\int \frac{dx}{\sqrt{2x - x^2}}$

ii) $\int \frac{(5x-3)dx}{(x+1)(x-3)}$ **(6 Marks)**

8. (a) Solve: $x\frac{dy}{dx} + y = x^3 y^6$ **(7 Marks)**

(b) Solve: $(x^4 - 2xy^2 + y^4)dx - (2x^2 y - 4xy^3 + siny)dy = 0$ **(7 Marks)**

(c) Solve: $xy\frac{dy}{dx} = 1 + x + y + xy$ **(6 Marks)**

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NEW SCHEME

MCA11A

USN

First Semester M.C.A Degree Examination, July/August 2004

Master of Computer Applications

Mathematics

Time: 3 hrs.] [Max.Marks : 100

Note: *Answer any FIVE full questions choosing at least TWO questions from each part.*

PART - A

1. (a) If $tan\ \theta = 2/3$, find the value of $sin2\theta - cos2\theta$. **(6 Marks)**

   (b) Find the real and imaginary parts of log (x+iy). **(7 Marks)**

   (c) Show that :

   $$2^7\ sin^8\theta = cos8\theta - 8cos6\theta + 28cos4\theta - 56cos2\theta + 35$$ **(7 Marks)**

2. (a) If $x = cos\alpha + i\ sin\alpha$ and $y = cos\beta + i\ sin\beta$

   Prove that $\frac{x-y}{x+y} = i\ tan\left(\frac{\alpha-\beta}{2}\right)$ **(6 Marks)**

   (b) If $sin\alpha + sin\beta + sin\gamma = cos\alpha + cos\beta + cos\gamma = 0$

   Prove that :

   i) $cos3\alpha + cos3\beta + cos3\gamma = 3cos(\alpha + \beta + \gamma)$

   ii) $sin3\alpha + sin3\beta + sin3\gamma = 3sin(\alpha + \beta + \gamma)$. **(8 Marks)**

   (c) Find all values of $\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)^{3/4}$

   What is the continued product of these values? **(6 Marks)**

3. (a) Find the rank of the matrix

   $$\begin{bmatrix} 2 & 3 & 4 & 5 \\ -2 & 3 & 2 & 1 \\ 2 & 3 & 4 & 6 \end{bmatrix}$$

   using elementary transformations. **(4 Marks)**

   (b) Test for consistency and solve

   $$x + 2y + 3z = 6$$
   $$2x + y + 4z = 7$$
   $$3x + 2y + 3z = 8$$ **(8 Marks)**

   (c) Find the inverse of the matrix

   $$\begin{bmatrix} -1 & -2 & 3 \\ -2 & 1 & 1 \\ 4 & -5 & 2 \end{bmatrix}$$

   using Caley - Hamilton theorem. **(8 Marks)**

Contd.... 2

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4. (a) Test for convergence the series

i) $\sum_{n=1}^{\infty} \frac{n+1}{2n^3+3n-2}$

ii) $\sum_{n=1}^{\infty} \frac{n!2^n}{n^n}$ **(8 Marks)**

(b) Test for convergence the series

$\frac{x}{1+\sqrt{1}} + \frac{x^2}{2+\sqrt{2}} + \frac{x^3}{3+\sqrt{3}} + ...to\ \infty\ x > 0.$ **(6 Marks)**

(c) Define absolute and conditional convergence. Show that the series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + ...to\ \infty$ is conditionally convergent. **(6 Marks)**

PART - B

5. (a) Find the angle between the curves $r = a(1 + cos\theta)$ and $r = a(1 - cos\theta)$. **(8 Marks)**

(b) Find n^{th} derivative of $\left[\frac{x+1}{x+2} + log\left(\frac{x+1}{x+2}\right)\right]$ **(6 Marks)**

(c) Evaluate $\lim_{x\to 0}\left(\frac{a^x+b^x}{2}\right)^{\frac{1}{x}}$ **(6 Marks)**

6. (a) If $u = log(tanx + tany)$ prove that

$sin\ 2x\frac{\partial u}{\partial x} + sin\ 2y\frac{\partial u}{\partial y} = 2$ **(6 Marks)**

(b) If $u = tan^{-1}\left[\frac{x^2+y^2}{x+y}\right]$ show that

$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}sin2u$ **(6 Marks)**

(c) If $u = f(x - y, y - z, z - x)$ prove that

$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$ **(8 Marks)**

7. (a) Evaluate: $\int_0^a x^4(a^2 - x^2)^{5/2}dx$. **(6 Marks)**

(b) Evaluate $\int_0^{\infty} \frac{x\,dx}{(1+x)(1+x^2)}$ **(6 Marks)**

(c) Evaluate $\iint_A xy\ dx\ dy$ where A is the domain bounded by x-axis, ordinates $x = 2a$ and the curve $x^2 = 4ay$. **(8 Marks)**

8. (a) Solve $\frac{dy}{dx} = \frac{y}{x} + sin(\frac{y}{x})$. **(6 Marks)**

(b) Solve $\frac{dy}{dx} - \frac{tany}{1+x} = (1 + x)e^x secy$. **(8 Marks)**

(c) Solve $(5x^4 + 3x^2y^2 - 2xy^3)dx + (2x^3y - 3x^2y^2 - 5y^4)dy = 0$. **(6 Marks)**

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NEW SCHEME

MCA11A

USN | | | | | | | | | | |

First Semester M.C.A Degree Examination, January/February 2005

Master of Computer Applications
(New Scheme)
Mathematics

Time: 3 hrs.] [Max.Marks : 100

Note: *1. Answer any FIVE full questions choosing at least TWO questions from each part.*
2. All questions carry equal marks.

PART - A

1. (a) Prove that

$$(1+\cos\theta+i\sin\theta)^n+(1+\cos\theta-i\sin\theta)^n=2^{n+1}\cos^n(\theta/2)\cos(n\theta/2)$$ **(6 Marks)**

(b) Find the real and imaginary parts of $\tan(x+iy)$ **(7 Marks)**

(c) Show that

$2^7\cos^8\theta=\cos 8\theta+8\cos 6\theta+28\cos 4\theta+56\cos 2\theta+35.$ **(7 Marks)**

2. (a) Prove that $\log\left(\frac{a+ib}{a-ib}\right)=2i\tan^{-1}(b/a)$ **(6 Marks)**

(b) Use De Moivre's theorem to solve the equation $x^5+1=0$ **(7 Marks)**

(c) If $\cosh(u+iv)=x+iy$, prove that

i) $\dfrac{x^2}{\cosh^2 u}+\dfrac{y^2}{\sinh^2 u}=1$

ii) $\dfrac{x^2}{\cos^2 v}-\dfrac{y^2}{\sin^2 v}=1$ **(7 Marks)**

3. (a) Find the rank of the matrix $\begin{bmatrix}1&2&3&2\\2&3&5&1\\1&3&4&5\end{bmatrix}$ using elementary row operations. **(6 Marks)**

(b) Test for consistency and solve the system :

$$x+y+z=6$$
$$x-y+2z=5$$
$$3x+y+z=8$$ **(7 Marks)**

(c) Find the eigen values and the eigen vector corresponding to the largest eigen value of the matrix

$$\begin{bmatrix}8&-6&2\\-6&7&-4\\2&-4&3\end{bmatrix}$$ **(7 Marks)**

Contd.... 2

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4. (a) Test for convergence of the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1}+\sqrt{n}}$ **(6 Marks)**

(b) Discuss the nature of the series $\frac{x}{1.2} + \frac{x^2}{2.3} + \frac{x^3}{3.4} + - - - - - -$ **(7 Marks)**

(c) Define absolute and conditional convergence. Show that the series $1 - \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} - \frac{1}{4\sqrt{4}} + - - - - - -$ is absolutely convergent. **(7 Marks)**

PART - B

5. (a) Find the n^{th} derivative of $cosx\ cos\ 2xcos3x$. **(6 Marks)**

(b) Prove that the curves $r = a(1 + cos\theta)$ and $r = b(1 - cos\theta)$ intersect at right angles. **(7 Marks)**

(c) Evaluate :

i) $\underset{x \to 0}{Lt} \frac{xe^x - log(1+x)}{x^2}$

ii) $\underset{x \to \frac{\pi}{2}}{lim} (1 - sinx) \times tanx$. **(7 Marks)**

6. (a) If $u = log(x^3 + y^3 + z^3 - 3xyz)$, prove that

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}$$ **(6 Marks)**

(b) If $u = tan^{-1}\left(\frac{x^3+y^3}{x+y}\right)$, Prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = sin2u$$ **(7 Marks)**

(c) If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$, show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$$ **(7 Marks)**

7. (a) Evaluate :

i) $\int x\ logx\ dx$ ii) $\int \frac{e^{sin^{-1}x}}{\sqrt{1-x^2}} dx$ **(6 Marks)**

(b) Evaluate $\int_0^a \frac{x^7\ dx}{\sqrt{a^2-x^2}}$ **(7 Marks)**

(c) Evaluate : $\int_0^a \int_0^x \frac{x}{x^2+y^2}\ dy\ dx$ **(7 Marks)**

8. (a) Solve : $\frac{dy}{dx} = \frac{y}{x} + tan\left(\frac{y}{x}\right)$ **(6 Marks)**

(b) Solve : $(x + 2y^3)\frac{dy}{dx} = y$ **(7 Marks)**

(c) Solve : $(x^2 + 4xy - 2y^2)dx + (y^2 - 4xy - 2x^2)dy = 0$ **(7 Marks)**

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MCA11A

USN

NEW SCHEME

First Semester M.C.A. Degree Examination, July 2006
Master of Computer Application
Mathematics

Time: 3 hrs.] [Max. Marks:100

Note: *1. Answer any FIVE full questions, choosing at least TWO questions from each part.*

PART - I

1 a. If sec A + tan A = a, then prove that $\sin A = \frac{a^2-1}{a^2+1}$. **(07 Marks)**

b. Find all the values of $(1+i)^{\frac{1}{3}}$. **(07 Marks)**

c. If $\sin(\alpha + i\beta) = x + iy$, show that $x^2 \sec h^2\beta + y^2 \cos ech^2\beta = 1$. **(06 Marks)**

2 a. Prove that $(1+\cos\theta + i\sin\theta)^n + (1+\cos\theta - i\sin\theta)^n = 2^{n+1}\cos^n\frac{\theta}{2}\cos\left(\frac{n\theta}{2}\right)$. **(07 Marks)**

b. Prove that $\cos 6\theta = 32\cos^6\theta - 48\cos^4\theta + 18\cos^2\theta - 1$. **(07 Marks)**

c. Separate the real and imaginary parts of $\tan^{-1}(x+iy)$. **(06 Marks)**

3 a. Find the rank of the matrix by elementary row operations:

$$\begin{pmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{pmatrix}$$ **(07 Marks)**

b. Test for consistency and hence solve:

$$3x + y + 2z = 3$$
$$2x - 3y - z = -3$$
$$x + 2y + z = 4$$ **(07 Marks)**

c. Find all the eigen values and the eigen vector corresponding to the least eigen value of the matrix:

$$\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$$ **(06 Marks)**

4 a. State the following:

i) Comparison Test.
ii) D' Alembert's Ratio Test.
iii) Raabe's Test. **(06 Marks)**

Contd... 2

b. Discuss the nature of the series:
$1+\frac{2^2}{2!}+\frac{3^2}{3!}+\frac{4^2}{4!}+\ldots\ldots\ldots\ldots\ldots$ **(07 Marks)**

c. Test the series:
$1-\frac{1}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}-\frac{1}{4\sqrt{4}}+-\ldots\ldots\ldots\ldots\ldots$
for : i) Convergence ii) Absolute Convergence. **(07 Marks)**

PART - B

5 a. If $y=\frac{x^3}{(x-1)(x-2)}$, then find y_n. **(07 Marks)**

b. If $y=\sin(m\sin^{-1}x)$, then prove that :
i) $(1-x^2)y_2-xy_1+m^2y=0$
ii) $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-(n^2-m^2)y_n=0$ **(07 Marks)**

c. Find the angle of intersection of the cardioides $r=a(1+\cos\theta)$ and $r=b(1-\cos\theta)$. **(06 Marks)**

6 a. If $u=\tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$. Prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\sin 2u$. **(07 Marks)**

b. If $u=x^2+y^2+z^2$ where $x=e^t$, $y=e^t\sin t$ and $z=e^t\cos t$. Show that $\frac{du}{dt}=4e^{2t}$. **(07 Marks)**

c. If $H=f(y-z, z-x, x-y)$.Then prove that $\frac{\partial H}{\partial x}+\frac{\partial H}{\partial y}+\frac{\partial H}{\partial z}=0$. **(06 Marks)**

7 a. Evaluate $\int\frac{2x+3}{x^2+x-30}dx$. **(07 Marks)**

b. Evaluate $\int\frac{2x+5}{\sqrt{x^2-2x+10}}dx$. **(07 Marks)**

c. Evaluate $\int_0^{\pi}\frac{dx}{5+4\cos x}$. **(06 Marks)**

8 Solve the following differential equations:

a. $(x^2+2y^2)dx-xydy=0$ when x = 1, y = 0. **(07 Marks)**

b. $\left(\frac{2xy+1}{y}\right)dx+\frac{y-x}{y^2}dy=0$. **(06 Marks)**

c. $(1-x^2)\frac{dy}{dx}-xy=1$. **(07 Marks)**

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MCA11A

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NEW SCHEME

First Semester MCA Degree Examination, Dec. 06 / Jan. 07

Mathematics

Time: 3 hrs.] [Max. Marks:100

Note: ***Answer any FIVE full questions, choosing at least TWO questions from each part.***

PART A

1 a. If $\tan\theta = \frac{2}{5}$ and $0 < \theta < 90^0$, find the value of $\frac{5\cos\theta + 2\sin\theta}{5\cos\theta - 2\sin\theta}$. **(06 Marks)**

b. State De Moivre's theorem. Use it to prove that:
$(1 + \mathrm{Cos}\theta + i\mathrm{Sin}\theta)^n + (1 + \mathrm{Cos}\theta - i\mathrm{Sin}\theta)^n = 2^{n+1}\mathrm{Cos}^n(\theta/2)\mathrm{Cos}(n\theta/2)$. **(07 Marks)**

c. Find all the values of :
$$\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)^{3/4}$$ **(07 Marks)**

2 a. If $\mathrm{Cosh}(A + iB) = x + iy$, show that: $x^2\mathrm{Sec}^2 A - y^2\mathrm{Cosec}^2 A = 1$. **(06 Marks)**

b. Expand $\mathrm{Cos}^8\theta$ in a series of cosines of multiples of θ. **(07 Marks)**

c. Separate the real and imaginary parts of $\tan(x + iy)$. **(07 Marks)**

3 a. Find the rank of the matrix by elementary row operations:
$$\begin{pmatrix} 1 & 0 & 2 & -2 \\ 2 & -1 & 0 & -1 \\ 1 & 0 & 2 & -1 \\ 4 & -1 & 3 & -1 \end{pmatrix}$$ **(06 Marks)**

b. Test for consistency and hence solve:
$$3x + y + 2z = 3$$
$$2x - 3y - z = -3$$
$$x + 2y + z = 4$$ **(07 Marks)**

c. Find all the eigen values and the eigen vectors corresponding to the largest eigen value of the matrix:
$$\begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix}$$ **(07 Marks)**

Contd.... 2

4 a. Test for the convergence or divergence of the series:
$$\frac{1}{1.2.3}+\frac{1}{2.3.4}+\frac{1}{3.4.5}+\ldots\ldots\ldots$$ **(06 Marks)**

b. Find the nature of the series:
$$1+\frac{x}{2}+\frac{x^2}{5}+\frac{x^3}{10}+\ldots\ldots..(x>0).$$ **(07 Marks)**

c. State Leibnitz's test for alternating series. Use it to discuss the nature of:
$$1-\frac{1}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}-\frac{1}{4\sqrt{4}}+\ldots\ldots\ldots$$ **(07 Marks)**

PART B

5 a. Find the n^{th} derivatives of:

i) Sinh4x ii) $\log\left(\frac{x^2-9}{2x+5}\right)$. **(06 Marks)**

b. If y = aCos(logx) + bSin(logx), prove that $x^2y_{n+2}+(2n+1)xy_{n+1}+(n^2+1)y_n=0$. **(07 Marks)**

c. Show that the curves $r^m=a^m\text{Cos}m\theta$ and $r^m=b^m\text{Sin}m\theta$ intersect at right angles. **(07 Marks)**

6 a. If $u=\log\left(\frac{x^4+y^4}{x+y}\right)$, show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=3$. **(06 Marks)**

b. If $u=\tan^{-1}\left(\frac{y}{x}\right)$, where $x=e^t-e^{-t}$ and $y=e^t+e^{-t}$, find $\frac{du}{dt}$. **(07 Marks)**

c. If z = f(x, y), and x = u − v, y = uv, prove that $(u+v)\frac{\partial z}{\partial y}=\frac{\partial z}{\partial u}+\frac{\partial z}{\partial v}$. **(07 Marks)**

7 a. Evaluate: $\int\frac{dx}{2x^2-2x+1}$. **(06 Marks)**

b. Evaluate: $\int\frac{\text{Sin}x-\text{Cos}x}{\sqrt{(\text{Sin}2x)}}dx$. **(07 Marks)**

c. Evaluate: $\int_0^a\frac{xdx}{\sqrt{\left(\frac{a^2-x^2}{a^2+x^2}\right)}}$. **(07 Marks)**

8 a. Solve: $(x^2-xy+y^2)dx-xydy=0$. **(06 Marks)**

b. Solve: $ydx+(3x-xy+2)dy=0$. **(07 Marks)**

c. Solve: $(x^2+y^3+6x)dx+y^2xdy=0$. **(07 Marks)**

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MCA11A

USN

NEW SCHEME

First Semester M.C.A Degree Examination, July 2007

Mathematics

Time: 3 hrs.] [Max. Marks:100

Note : ***Answer any FIVE full questions choosing at least two questions from each part.***

Part A

1 a. Use De-Moivre's theorem to prove that,

$$\left(\frac{1+\sin\alpha+i\cos\alpha}{1+\sin\alpha-i\cos\alpha}\right)^n=\cos\left(\frac{n\pi}{2}-n\alpha\right)+i\sin\left(\frac{n\pi}{2}-n\alpha\right)$$ **(07 Marks)**

b. Prove that $2^5\sin^4\theta\cos^2\theta=\cos6\theta-2\cos4\theta-\cos2\theta+2$ **(07 Marks)**

c. Use De Moivre's theorem to solve $x^5+1=0$ **(06 Marks)**

2 a. If $p=\cos\theta+i\sin\theta$, $q=\cos\phi+i\sin\phi$, show that

$$\frac{p-q}{p+q}=i\tan\left(\frac{\theta-\phi}{2}\right).$$ **(06 Marks)**

b. Find real and imaginary parts of the function $\tan^{-1}(\alpha+i\beta)$. **(08 Marks)**

c. Prove that $\tan\left[i\log\left(\frac{a-ib}{a+ib}\right)\right]=\frac{2ab}{a^2-b^2}$ **(06 Marks)**

3 a. Using elementary transformations find the rank of the matrix $\begin{pmatrix}1&2&3&-2\\2&-2&1&3\\3&0&4&1\end{pmatrix}$ **(06 Marks)**

b. Find the Eigen values and Eigen vector corresponding to largest Eigen value of the matrix $\begin{pmatrix}5&4&-4\\4&5&-4\\-1&-1&2\end{pmatrix}$. **(07 Marks)**

c. State Caley-Hamilton theorem and use it to find A^{-1}, given $A=\begin{pmatrix}1&2&4\\-1&0&3\\3&1&-2\end{pmatrix}$. **(07 Marks)**

4 a. Examine the convergence of the series,

$$\frac{1}{2.3}+\frac{1.3}{2.4.5}+\frac{1.3.5}{2.4.6.7}+\cdots\cdots\infty$$ **(07 Marks)**

b. Examine the convergence of the series,

$$\sum_{n=1}^{\infty}\frac{(n!)^2}{(2n)!}x^n \text{ where } x>0$$ **(08 Marks)**

c. Examine the convergence of the series,

$$\sum\left[\sqrt{n^2+1}-n\right]$$ **(05 Marks)**

Contd.... 2

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Part B

5 a. If $x^y = y^x$ prove that $\dfrac{dy}{dx} = \dfrac{y(y - x\log y)}{x(x - y\log x)}$. **(07 Marks)**

b. If $y = a\cos(\log x) + b\sin(\log x)$, show that $x^2 y_{n+2} + x(2n+1)y_{n+1} + (n^2+1)y_n = 0$ **(07 Marks)**

c. Find angle between the curves $r = a(1+\sin\theta)$ and $r = a(1-\sin\theta)$ **(06 Marks)**

6 a. If $u = \dfrac{1}{\sqrt{x^2+y^2+z^2}}$, prove that $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} + \dfrac{\partial^2 u}{\partial z^2} = 0$. **(06 Marks)**

b. If $u = f(y-z,\ z-x,\ x-y)$, show that $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} + \dfrac{\partial u}{\partial z} = 0$. **(06 Marks)**

c. State Euler's theorem on homogeneous functions. Use it to show that if $u = \log\left(\dfrac{x^2+y^2}{x+y}\right)$ then $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = 1$. **(08 Marks)**

7 a. Evaluate $\displaystyle\int \frac{dx}{(x+2)\sqrt{x^2+6x+1}}$. **(07 Marks)**

b. Evaluate $\displaystyle\int \frac{dx}{2\cos x + 3\sin x - 1}$. **(06 Marks)**

c. Evaluate $\displaystyle\int_0^a \int_0^{\sqrt{a^2-x^2}} x^2 y\,dy\,dx$. **(07 Marks)**

8 Solve the following differential equations :

a. $x\,dy - y\,dx = \sqrt{x^2+y^2}\,dx$. **(06 Marks)**

b. $\dfrac{dy}{dx} = \dfrac{x^2+3y^2}{2x^2+3xy}$. **(07 Marks)**

c. $(1+x)\dfrac{dy}{dx} - \tan y = (1+x)^2 e^x \sec y$ **(07 Marks)**

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07MCA11B

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First Semester MCA Degree Examination, Dec. 07 / Jan. 08
Mathematics

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, choosing at least TWO questions from each part.***

PART – A

1 a. If $\sec A + \tan A = a$, prove that $\sin A = \frac{a^2 - 1}{a^2 + 1}$ **(07 Marks)**

b. If $A + B = 45^0$, then show that $(1 + \tan A)(1 + \tan B) = 2$. Hence deduce the value of $\tan^{-1} 22\frac{1}{2}^0$. **(07 Marks)**

c. With the usual notations, prove that $\sin(A + B) = \sin A \cos B + \cos A \sin B$. **(06 Marks)**

2 a. Prove that $(1 + \cos\theta + i\sin\theta)^n + (1 + \cos\theta - i\sin\theta)^n = 2^{n+1}.\cos^n \theta/2 .\cos\left(\frac{n\theta}{2}\right)$. **(07 Marks)**

b. Prove that $32\cos^6\theta = \cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10$. **(07 Marks)**

c. Using De'Moivre's theorem, solve the equation $x^4 - x^3 + x^2 - x + 1 = 0$. **(06 Marks)**

3 a. If $A = \begin{bmatrix} 1 & 3 & 0 \\ -1 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 2 & 3 \\ -1 & 1 & 2 \end{bmatrix}$ compute AB and BA. Show that $AB \neq BA$. **(07 Marks)**

b. Express the matrix A as the sum of a symmetric and a skew–symmetric matrix.

$A = \begin{bmatrix} 4 & 2 & -3 \\ 1 & 3 & -6 \\ -5 & 0 & -7 \end{bmatrix}$. **(07 Marks)**

c. Find the rank of matrix

$A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 0 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$. **(06 Marks)**

4 a. Test for consistency and hence solve

$3x + y + 2z = 3$
$2x - 3y - z = -3$
$x + 2y + z = 4$ **(07 Marks)**

b. Find the eigen values and eigen Vectors of the matrix

$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$. **(07 Marks)**

c. Using Cayley – Hamilton theorem, find the inverse of the matrix

$A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$. **(06 Marks)**

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PART – B

5 a. If $y = \cos x \cos 2x \cos 3x$, find y_n. **(07 Marks)**

b. If $y = \dfrac{1}{4x^2 + 8x + 3}$, find y_n. **(07 Marks)**

c. If $y = a\cos(\log x) + b\sin(\log x)$, prove that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$. **(06 Marks)**

6 a. Find the pedal equation of the curve $\dfrac{2a}{r} = 1 - \cos\theta$. **(07 Marks)**

b. Find the angle between the curves $r = \sin\theta + \cos\theta$ and $r = 2\sin\theta$. **(07 Marks)**

c. Evaluate $\lim\limits_{x \to 0} \dfrac{x\cos x - \log(1+x)}{x^2}$. **(06 Marks)**

7 a. Evaluate $\int \dfrac{2x+3}{x^2 + x - 30}\,dx$. **(07 Marks)**

b. Evaluate $\int \dfrac{dx}{\sqrt{1 + x - x^2}}$. **(07 Marks)**

c. Evaluate $\int_0^{\pi/2} \dfrac{\cos x}{1 + \sin^2 x}\,dx$. **(06 Marks)**

8 a. Solve $\dfrac{dy}{dx} = \dfrac{x(2\log x + 1)}{\sin y + y\cos y}$. **(07 Marks)**

b. Solve $(x^2 - y^2)dx = xydy$. **(07 Marks)**

c. Solve $(2xy + y - \tan y)dx + (x^2 - x\tan^2 y + \sec^2 y)dy = 0$. **(06 Marks)**

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MCA11A

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First Semester MCA Examination, June/July 08
Mathematics

Time: 3 hrs. Max. Marks:100

Note : *Answer any FIVE full questions.*

PART - A

1 a. If $\operatorname{cosec}\theta = \frac{13}{5}, \frac{\pi}{2} < \theta < \pi,$ find the value of $\frac{2\sin\theta - 3\cos\theta}{3\sin\theta + 2\cos\theta}$ **(06 Marks)**

b. If sin (A+iB)=x + iy ,show that $\frac{x^2}{\sin^2 A} - \frac{Y^2}{\sin^2 B} = 1$ **(07 Marks)**

c. Show that $\sin^7\theta = 7\sin\theta - 56\sin^3\theta + 112\sin^5\theta - 64\sin^7\theta$. **(07 Marks)**

2 a. Prove that $(1+\cos + i\sin\theta)^n + (1+\cos\theta - i\sin\theta)^n = 2^{n+1}\cos^n\frac{\theta}{2}\cos\frac{n\theta}{2}$. **(06 Marks)**

b. If $x = \cos\alpha + i\sin\alpha$, $y = \cos\beta + i\sin\beta$ and $z = \cos\nu + i\sin\nu$ and $x + y + z = 0$, show that $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$ **(07 Marks)**

c. Find all the values of $(-1)^{\frac{1}{6}}$. **(07 Marks)**

3 a. Find the rank of the matrix $\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ **(06 Marks)**

b. Are the equations $x + y + 2z = -1$, $x + 2y - 3z = 2$ and $2x + 5y - 2z = -2$ consistent? If so, solve them. **(07 Marks)**

c. Find the characteristic equation of the matrix

$$\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$$

Obtain its inverse using Cayley - Hamilton theorem. **(07 Marks)**

4 a. Examine the convergence of the infinite series $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \text{-------to}\infty$ **(06 Marks)**

b. Test for convergence of the series $\frac{1}{2\sqrt{1}} + \frac{x^2}{3\sqrt{2}} + \frac{x^4}{4\sqrt{3}} + \frac{x^6}{5\sqrt{4}} + \text{---- to } \infty$ **(07 Marks)**

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c. Show that the alternating series

$$1-\frac{1}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}-\frac{1}{4\sqrt{4}}+--------\text{to } \infty$$

is absolutely convergent. **(07 Marks)**

PART - B

5 a. Find the n^{th} derivative of $\frac{x+3}{x^2+x-2}$ **(06 Marks)**

b. Show that the curve $r^n = a^n \cos n\theta$ and $r^n = b^n \sin n\theta$ intersect orthogonally. **(07 Marks)**

c. Evaluate i) $\lim\limits_{x\to 0}\frac{e^x - e^{\sin x}}{x-\sin x}$ and ii) $\lim\limits_{x\to 0}(1+\sin x)^{\cot x}$ **(07 Marks)**

6 a. If $u=\left(x^2+y^2+z^2\right)^{-\frac{1}{2}}$ show that

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=0$$ **(06 Marks)**

b. If $u=\cos^{-1}\frac{x+y}{\sqrt{x}+\sqrt{y}}$, show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+\frac{1}{2}\cot u=0$ **(07 Marks)**

c. If V is a function of x and y where $x = r\cos\theta$ and $y = r\sin\theta$, show that

$$\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2=\left(\frac{\partial u}{\partial n}\right)^2+\frac{1}{r^2}\left(\frac{\partial u}{\partial \theta}\right)^2$$ **(07 Marks)**

7 a. Find the value of $\int_0^1 x^4\sqrt{1-x^2}\,dx$ **(06 Marks)**

b. Evaluate $\int\sqrt{\frac{\sin^{-1}x}{1-x^2}}\,dx$ **(07 Marks)**

c. Evaluate $\iint_R y^2\,dy\,dx$ where R is the region in the first quadrant bounded by the circle $x^2+y^2=1$. **(07 Marks)**

8 Solve

a. $\frac{dy}{dx}=(x+y+1)^2$ **(06 Marks)**

b. $\frac{dy}{dx}+x\sin 2y=2x\cos^2 y$ **(07 Marks)**

c. $\frac{dy}{dx}+\frac{y\cos x+\sin y+y}{\sin x+x\cos y+x}=0$ **(07 Marks)**

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07MCA11B

First Semester MCA Degree Examination, Dec.09/Jan.10
Mathematics

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

1 a. If $x = \cos\alpha + i\sin\alpha$ and $y = \cos\beta + i\sin\beta$
Prove that $\dfrac{x-y}{x+y} = i\tan\left(\dfrac{\alpha-\beta}{2}\right)$. **(07 Marks)**

b. If $\sin\alpha + \sin\beta + \sin\gamma = \cos\alpha + \cos\beta + \cos\gamma = 0$ then prove that
i) $\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos(\alpha+\beta+\gamma)$
ii) $\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3\sin(\alpha+\beta+\gamma)$ **(07 Marks)**

c. Prove that $\cos 6\theta = 32\cos^6\theta - 48\cos^4\theta + 18\cos^4\theta - 1$. **(06 Marks)**

2 a. Find all the roots of $(-1+i)^{2/5}$. **(07 Marks)**

b. If $2\cos\theta = x + \dfrac{1}{x}$ and $2\cos\phi = y + \dfrac{1}{y}$, show that one of the values of
i) $x^m y^n + \dfrac{1}{x^m y^n}$ is $2\cos(m\theta + n\phi)$
ii) $\dfrac{x^m}{y^n} + \dfrac{y^n}{x^m}$ is $2\cos(m\theta - n\phi)$. **(06 Marks)**

c. Use De' Moivre's theorem to solve the equation $x^9 - x^5 + x^4 - 1 = 0$. **(07 Marks)**

3 a. Find the rank of the matrix A, using the elementary row operations, where
$$A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ -1 & -2 & 6 & -7 \end{bmatrix}.$$ **(06 Marks)**

b. For what value of λ, will the following system have an unique solution?
$3x - y + \lambda z = 1,\ 2x + y + z = 2,\ x + 2y - z = -1$. **(07 Marks)**

c. Verify Cayley – Hamilton theorem and compute A^{-1} and A^4 for the matrix
$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}.$$ **(07 Marks)**

4 a. Find the n^{th} derivative of $e^x \cos x \cos 2x$. **(06 Marks)**

b. Find $\dfrac{dy}{dx}$ if $x^y + y^x = 1$. **(07 Marks)**

c. If $y = e^{m\sin^{-1}x}$ show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0$. **(07 Marks)**

5 a. Find the angle of intersection of the curves $r = a(1 - \cos\theta)$ and $r = 2a\cos\theta$. **(07 Marks)**

b. Find the pedal equation of the curves $r^m = a^m \cos m\theta$. **(06 Marks)**

c. Evaluate $\underset{x \to 2}{\text{Lt}} \left(\frac{1}{x-2} - \frac{1}{\log(x-1)} \right)$. **(07 Marks)**

6 a. Evaluate $\underset{x \to 0}{\text{Lt}} (\cos x)^{1/x^2}$. **(07 Marks)**

b. Evaluate i) $\int_0^{\pi} x \sin^4 x \cos^6 x \, dx$.

ii) $\int_0^1 x^4 (1-x)^{3/2} \, dx$. **(06 Marks)**

c. Evaluate $\int \frac{1}{4 + 5\cos x} dx$. **(07 Marks)**

7 a. Evaluate $\int \frac{(2x+3)}{x^2 + x + 1} dx$. **(07 Marks)**

b. Evaluate $\int \frac{1}{(x+2)\sqrt{x+3}} dx$. **(07 Marks)**

c. Evaluate $\int \frac{dx}{2x^2 - 2x + 1}$. **(06 Marks)**

8 a. Solve $(y^2 + xy^2)\,dx + (x^2 - yx^2)\,dy = 0$. **(06 Marks)**

b. Solve $x\frac{dy}{dx} - y = \sqrt{x^2 + y^2}$. **(07 Marks)**

c. Solve $\frac{dy}{dx} + \frac{3x^2 y}{1 + x^3} = \frac{\sin^2 x}{1 + x^3}$. **(07 Marks)**

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07MCA11B

First Semester MCA Degree Examination, May/June 2010
Mathematics

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. If $\tan A + \sin A = m$ and $\tan A - \sin A = n$, then prove that $(m^2 - n^2)^2 = 16mn$. **(06 Marks)**

b. If A = 45°, verify that $\sin 2A = \dfrac{2\tan A}{1+\tan^2 A}$. **(07 Marks)**

c. With usual notations, prove that $\cos(A + B) = \cos A \cos B - \sin A \sin B$. **(07 Marks)**

2 a. If $\sin(A + iB) = x + iy$, prove that $x^2 \operatorname{cosec}^2 A - y^2 \sec^2 A = 1$. **(06 Marks)**

b. Using De Moivre's theorem, solve the equation $x^5 + 1 = 0$. **(07 Marks)**

c. Prove that $(1+\cos\theta - i\sin\theta)^n + (1+\cos\theta - i\sin\theta)^n = 2^{n+1}\cos^n\left(\dfrac{\theta}{2}\right)\cos\left(\dfrac{n\theta}{2}\right)$. **(07 Marks)**

3 a. If $A = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 0 & -2 & 3 \end{bmatrix}$,

verify that A(B+C) = AB+AC. **(06 Marks)**

b. Determine the roots of the matrix $\begin{bmatrix} 3 & 5 & 7 & 8 & 4 \\ -1 & 2 & 3 & 1 & 3 \\ 4 & 5 & 1 & 2 & -1 \\ 1 & -5 & 6 & -7 & -8 \end{bmatrix}$ **(07 Marks)**

c. Express the matrix A as the sum of symmetric and skew-symmetric matrices

$$A = \begin{bmatrix} 4 & 2 & -3 \\ 1 & 3 & -6 \\ -5 & 0 & -7 \end{bmatrix}$$ **(07 Marks)**

4 a. Test the consistency and solve the system
$x + y + z = 6$, $x - y + 2z = 5$ and $3x + y + z = 8$. **(06 Marks)**

b. Find the eigen value and the eigen vector corresponding to the least eigen value of the matrix $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$. **(07 Marks)**

c. Using the Cayley-Hamilton theorem, find the inverse of the matrix $\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$. **(07 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

5 a. Find the nth derivative of $\frac{x}{6x^2 - x - 2}$. **(06 Marks)**

b. If $y = \sin(m \sin^{-1}x)$, prove that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 - m^2)y = 0$. **(07 Marks)**

c. Find the nth derivative of $e^{ax} \cos(bx + c)$. **(07 Marks)**

6 a. Find the angle of intersection between two curves
$r = a\sec^2\frac{\theta}{2}$ and $r = b\operatorname{cosec}^2\frac{\theta}{2}$. **(06 Marks)**

b. Find the pedal equation of the curve $l/r = 1 + e\cos\theta$. **(07 Marks)**

c. Evaluate (i) $\lim_{x \to 0} \frac{xe^x - \log(1+x)}{x^2}$ (ii) $\lim_{x \to \pi/2} (1 - \sin x)\tan x$ **(07 Marks)**

7 a. Evaluate (i) $\int x \log x \, dx$ (ii) $\int \frac{e^{\sin^{-1}x}}{\sqrt{1-x^2}} dx$ **(06 Marks)**

b. Evaluate $\int_0^a \frac{x^7}{\sqrt{a^2 - x^2}} dx$. **(07 Marks)**

c. Evaluate $\int_0^\pi \frac{dx}{5 + 4\cos x} dx$. **(07 Marks)**

8 a. Solve $\left(\frac{2xy+1}{y}\right)dx + \frac{y-x}{y^2}dy = 0$. **(06 Marks)**

b. Solve $(3y + 2x + 4)dx - (4x + 6y + 5)dy = 0$. **(07 Marks)**

c. Solve $(x + 2y^3)\frac{dy}{dx} = y$. **(07 Marks)**

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